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METHODOLOGICAL FEATURES OF INVESTIGATING SETS HAVING THE CARDINALITY OF A CONTINUUM

Students of mathematical specialties in higher education institutions encounter Cantor sets while studying the structure of closed sets in courses on function theory and functional analysis. In classical textbooks, the ternary numeral system is used to determine the cardinality of these sets. This approach establishes that the Cantor set has the cardinality of the continuum.

This paper presents a method for determining the cardinality of Cantor sets using alternative numeral systems. This approach integrates the study of Cantor sets in function theory and functional analysis with numeral systems covered in algebra and programming courses. It significantly increases the number of practical tasks for students' independent work on Cantor sets, deepens their understanding of the techniques for constructing such sets, and enhances their mastery of methods for determining their cardinality.

Certain elements of this work can be utilized in extracurricular mathematics activities or for club-based projects in specialized classes with an advanced focus on mathematics in general secondary education institutions.

Keywords: *methodological features; number system; set; cardinality of set; Cantor set; metric space; fractals; functional analysis.*

Statement of the problem. In methodological and educational literature, Cantor sets are often presented as examples of sets with the cardinality of the continuum. Their cardinality is classically determined using the ternary numeral system by establishing a one-to-one correspondence between elements of the Cantor set and their representations in this numeral system.

When students delve into the method for determining the cardinality of Cantor sets, they often question the necessity of using the ternary numeral system specifically. This paper demonstrates that other numeral systems can also be used without significantly complicating the proof of the corresponding theorem on the cardinality of the Cantor set. An example of constructing a Cantor set and the corresponding Cantor function based on the quinary numeral system is provided.

If we analyze the mathematical foundation necessary for studying number systems on a perfect set, we will find the following topics: the concept of a segment and its division into parts, actions on ordinary fractions, elements of combinatorics, the concept of a set and actions on it, geometric progression and the sum of an infinitely decreasing geometric progression, inverse function, limit and its application, elements of mathematics methodology. Since these topics are studied in the school mathematics course, for students of senior grades of general secondary education institutions who study mathematics at an advanced level, acquaintance with Cantor sets and fractal structures is also possible, as a rule, by means of informal education.

Purpose of the article. The purpose of this work is to study sets similar to the traditional perfect Cantor set and the Cantor function, followed by the justification and development of a plan to integrate the topic "Cantor Set and Numeral Systems" into the course on function theory and functional analysis.

Overview of results relevant to the topic of the article. The concepts of open and closed sets, set cardinality, and metric spaces are fundamental notions in function theory and functional analysis. All results of this theory are based on these concepts, and they are applied in many other branches of mathematics. In particular, Cantor sets, their structure, visualization, and construction methods are used in the study of fractal structures and their development using information technologies. In particular, in (Rushton, 2020), the impact of the orthodox foundation of mathematics, based on Cantor's informal set theory, on the study of mathematics and computer science is discussed, along with an analysis of its shortcomings.

Georg Cantor's views on set theory and the concept of infinity have significantly influenced modern mathematics, philosophy, and theology. In Jakub Gomułka (Gomułka, 2021), the contrasting perspectives of Georg Cantor and Ludwig Wittgenstein on set theory and the concept of infinity are compared. In Karim Zahidi (Zahidi, 2024), some philosophical aspects of mathematical practice are examined, including an evaluation of Ludwig Wittgenstein's critique of Cantor's set theory. In Roman Murawski (Murawski, 2021), Cantor's theological views and the influence of Catholic theology on his theory of infinity are discussed.

Cantor's set theory is extensively applied in contemporary mathematical research. In (Keqin Cui, et al., 2022), an algorithm is developed to identify all possible numbers of a geometric progression contained in certain Cantor sets. In Mykola Pratsiovytyi and Dmytro Karvatskyi (Pratsiovytyi, Karvatskyi, 2023), the properties of the set of partial sums of a convergent trigonometric series are studied. It is shown that, as the values of the argument vary within the interval (0;1), this set can become a Cantor-type set. Special attention is given to the geometric interpretation of constructing Cantor sets and the Cantor function, as this relates to the visualization of fractal sets. In Mykola Pratsiovytyi, Iryna Lysenko, Sofiia Ratushniak, Alexander Tsokolenko (Pratsiovytyi, et al., 2024), fractal lines are examined, which play a significant role in fractal geometry and have wide applications, particularly in physics and in modeling antennas. In this context, it is also worth noting the work (Kuz'mich, 2022), which addresses the geometric interpretation and visualization of fundamental geometric concepts during the initial introduction to metric spaces.

Cantor sets are used in solving complex physical and engineering problems. In particular, Edward Belbruno (Belbruno, 2024) explores the structure of the boundary region of weak stability for the planar restricted three-body problem concerning a secondary mass point. It has been established that, in certain cases, the boundary may consist of an infinite family of Cantor sets, thus exhibiting a fractal nature. In Roberto Paroni, Brian Seguin (Paroni, Seguin, 2024), the possibility of bending a one-dimensional linear-elastic beam only at the points of a Cantor set is investigated. It has been shown that, for a sequence of beams, each capable of bending only at a set associated with a specific step in the construction of a Cantor set, the resulting energy in the limit exhibits a structure similar to that of traditional bending energy.

Methods. When establishing the results of the study, the authors used the method of analyzing previous publications on the methodology of individual sections of functional analysis and set theory in higher education institutions. The method of analytical transformations based on the representation of real numbers in different number systems was also used. When constructing the Cantor set, the classical method of extracting non-intersecting intervals from a unit segment was used.

Research results. Let us outline some fundamental concepts and facts from set theory that will be referenced in this work. We will use the concept of the cardinality of a set.

Two sets are said to have the same cardinality if there exists a rule establishing a one-to-one correspondence between their elements, i.e., if the two sets are equivalent. The concept of cardinality generalizes the idea of the number of elements in a set. Specifically, for a finite set, these two notions coincide. A set equivalent to the set of natural numbers is said to have a countable number of elements and is referred to as countable. If a set is equivalent to the set of points in the segment [0; 1], it is said to have the cardinality of the continuum.

The classical Cantor set is constructed from the set of all points in the segment [0; 1] through an iterative and infinite process of removing intervals, where each removed interval is one-third the length of the segment from which it is removed. The resulting set of points in the segment [0; 1] is called the Cantor set. A notable feature of this set is that it is discontinuous everywhere on the segment [0; 1], meaning it contains no segment of any length, however small. Moreover, it is a perfect set, meaning it consists of all its limit points; in other words, it is a closed set without isolated points. The classical proof that the Cantor set has the cardinality of the continuum relies on a method that associates each point in the segment with a sequence representing the number in the ternary numeral system. From this, it follows that the Cantor set has the cardinality of the continuum.

The concept of the Cantor set is closely related to the Cantor function. This function is defined on the segment [0; 1]. It is continuous, non-decreasing, and its derivative is zero at every point of any adjacent interval, that is, almost everywhere. The Cantor function is also known as the jump function at the points of the Cantor set.

Let us construct a Cantor set using a scheme slightly different from the classical one. In this construction, we will rely on the representation of numbers in a numeral system with base q, where $q=2n+1$ ($n\in N$).

Consider the unit segment $F_{0q}=[0; 1]$. Divide it into $q=2n+1$equal parts, where n is some fixed natural number. Let us agree that the numbering of the segments will start with the digit 0, meaning that the segment with ordinal number m corresponds to the number $m-1$, where $m\in\{1, 2, \ldots, 2n+1\}$. Now, we remove all intervals corresponding to odd numbers $2m+1$, where $m\in\{1, 2, \ldots, n\}$. The remaining segments are then combined, resulting in:

$$F_{1q}=[0; \frac{1}{2n+1}]\cup[\frac{2}{2n+1}; \frac{3}{2n+1}]\cup[\frac{2}{2n+1}; 1]\cdot$$

Similarly, each segment of the set F_{1q} is divided again into $2n+1$ equal parts, and the intervals corresponding to the odd numbers $2m+1$, where $m\in\{1, 2, \ldots, n\}$, are removed. Note that the first segment corresponds to the number 0. The intersection of these segments is denoted as F_{2q}.

We continue this process indefinitely and define: $F_q=\bigcap_{k=0}^{\infty}F_{kq}$.

It can be proven that the set F_q, formed through this process, is a perfect Cantor set. Each number x from the segment [0; 1] can be represented in the corresponding numeral system with base $q=2n+1$ ($n\in N$) as:

$$x=\frac{a_1}{(2n+1)}+\frac{a_2}{(2n+1)^2}+\ldots+\frac{a_P}{(2n+1)^p}+\ldots,$$

where $a_p\in\{0, 1, 2, \ldots, 2n\}$.

It can be proven that each point $x\in F_q$ corresponds to a sequence of numbers: a_1, a_2, …, a_p, …, where a_p can only take even values $2m$, where $m\in\{0,1, \ldots, n\}$.

To find the total length of the removed intervals.

For F_{1q}, the total length F_{1q} of the removed intervals is:

$$d_{1q}=n\cdot\frac{1}{2n+1}=\frac{n}{2n+1}.$$

For F_{2q}, the total length d_{2q} is:

$$d_{2q}=\frac{n}{2n+1}+(n+1)\cdot\frac{n}{(2n+1)^2}.$$

For F_{3q}, the total length d_{3q} is:

$$d_{3q}=\frac{n}{2n+1}+(n+1)\cdot\frac{n}{(2n+1)^2}+(n+1)^2\cdot\frac{n}{(2n+1)^3}$$

For F_{kq}, the total length d_{kq} is:

$$d_{3q}=\frac{n}{2n+1}+(n+1)\cdot\frac{n}{(2n+1)^2}+(n+1)^2\cdot\frac{n}{(2n+1)^3}+\ldots+ +(n+1)^{k-1}\cdot\frac{n}{(2n+1)^k}.$$

Using the formula for the sum of an infinite geometric series, we find that the sequence $\{d_{kq}\}$, representing the sums of the lengths of the removed intervals, converges to 1:

$$\lim_{k\to\infty}d_{kq}=\frac{\frac{n}{2n+1}}{1-\frac{n+1}{2n+1}}=\frac{n}{2n+1}\cdot\frac{2n+1}{n}=1.$$

Thus, the total length of the removed intervals equals 1, and therefore, the measure of the set F_q is 0.

A point x of a set A is called an isolated point of this set if there exists a neighborhood of x that contains no points of A other than x itself. Since the lengths of the segments comprising the sets F_{kq} approach zero as $k\to\infty$, for each point $x\in F_q$, at least the endpoints of some segments from the sets v converge to x, and therefore are contained in its neighborhood. Thus, the constructed set F_q has all the properties of a perfect set.

Since a perfect Cantor set can be constructed for any numeral system with base $q=2n+1$ ($n\in N$), we define the Cantor function $f_q(x)$ for each of these systems.

If $x\in F_q$, then x can be represented as:

$$x=\frac{a_1}{(2n+1)}+\frac{a_2}{(2n+1)^2}+\ldots+\frac{a_P}{(2n+1)^p}+\ldots, \qquad (1)$$

where the numbers a_p can take values of the form $2m$, where $m\in\{0, 1, \ldots, n\}$. The function $f_q(x)$ is defined by the equation:

$$f_q(x)=\frac{\frac{a_1}{2}}{(n+1)}+\frac{\frac{a_2}{2}}{(n+1)^2}+\ldots+\frac{\frac{a_P}{2}}{(n+1)^p}+\ldots,$$

hat is, we have determined the number corresponding to the sequence $\frac{a_1}{2}$; $\frac{a_1}{2}$; …; $\frac{a_1}{2}$; … in the numeral system with base $n+1$.

If $x\notin F_q$ then x belongs to one of the intervals removed at the k-th step. It can be proven that the values of the function $f_q(x)$ at the endpoints of such an interval are equal. Therefore, the value of the function at any point within this interval is considered equal to its value at either endpoint.

Let us illustrate this algorithm for constructing the Cantor function in the quinary numeral system, i.e., for $q=2\cdot2+1=5$ ($n+1=3$). In subsequent steps, equation (1) will be briefly written as:

$$x=(a_1, a_2, \ldots, a_p, \ldots)_q. \qquad (2)$$

Let us calculate the value of the function $f_5(x)$ on the interval $\left(\frac{1}{5};\frac{2}{5}\right)\cdot$ Choose one of the endpoints of the interval, for example, $x=\frac{2}{5}\cdot$ According to the notation in (2), we have: $\frac{2}{5}=(2,0,0,...)_5\cdot$ Dividing all coordinates in half, we get the ternary notation of the number: $(1, 0, 0, …)_3$. We calculate the value of the function $f_5(x)$ at the point $x=\frac{2}{5}$:

$$f_5\left(\frac{2}{5}\right)=\frac{1}{3}+\frac{0}{3^2}+\frac{0}{3^3}+...=\frac{1}{3}.$$

We make sure that at the other end of the interval $\frac{1}{5}=(0,4,4,...)_5$ the function $f_5(x)$ takes the same value. Divide the coordinates in half and get the ternary notation of the number: $(0, 2, 2, …)_3$. We calculate the value of the function at this point:

$$f_5\left(\frac{1}{5}\right)=\frac{0}{3}+\frac{2}{3^2}+\frac{2}{3^3}+...=\frac{\frac{2}{9}}{1-\frac{1}{3}}=\frac{2}{9}\cdot\frac{3}{2}=\frac{1}{3}.$$

Therefore, each point in the interval $\left(\frac{1}{5};\frac{2}{5}\right)$, corresponds to the value of the function $f_5(x)=\frac{1}{3}$.

Similarly, we calculate the value of the function $f_5(x)$ on the interval $\left(\frac{3}{5};\frac{4}{5}\right)\cdot$

We choose the point $x=\frac{4}{5}=(4,0,0,...)_5$. We divide the coordinates in half and get the ternary notation of the number: $(2, 0, 0, …)_3$. We calculate the value of $f_5(x)$ at the point $x=\frac{4}{5}$:

$$f_5\left(\frac{4}{5}\right)=\frac{2}{3}+\frac{0}{3^2}+\frac{0}{3^3}+...=\frac{2}{3}.$$

Let's calculate the value of the function at the point $x=\frac{3}{5}=(2,4,4,...)_5$. Divide the coordinates in half and get the ternary notation of the number: $(1, 2, 2, …)_3$. Let's calculate the value of the function $f_5(x)$ at the point $x=\frac{3}{5}$:

$$f_5\left(\frac{3}{5}\right)=\frac{1}{3}+\frac{2}{3^2}+\frac{2}{3^3}+...=\frac{1}{3}+\frac{\frac{2}{9}}{1-\frac{1}{3}}=\frac{1}{3}+\frac{2}{9}\cdot\frac{3}{2}=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}.$$

Therefore, each point in the interval $\left(\frac{3}{5};\frac{4}{5}\right)$ corresponds to the value of the function $f_5(x)=\frac{2}{3}$.

Using the same scheme, we calculate the value of the function on the interval $\left(\frac{1}{25};\frac{2}{25}\right)\cdot$ It is easy to verify that each point of this interval corresponds to the value of the function $f_5(x)=\frac{1}{9}$.

Continuing similar actions for each removed interval, we obtain all the values of the function $f_5(x)$ on the segment [0, 1].

The described research can be partially implemented in various types of informal mathematical education in secondary education institutions. Based on the ideas of mathematicians-methodologists, regulatory and legal documents, and understanding the importance of developing students' research skills, it is possible to propose a cross-curricular task on the topic "Number systems on perfect sets", which students will perform in the classes of the mathematical circle, or the corresponding elective during their studies in grades 5–11. Although this topic is studied in the course of higher mathematics, however, in our opinion, if we approach this issue gradually, the task is quite feasible for school-age students.

We propose to implement the study of elements of Cantor sets, using the basic mathematical concepts that students receive in the relevant classes, in the following sequence:

1. Introduction to number systems (grade 5).
2. Introduction to the concept of a segment and its division into parts (grade 5).
3. Formation of the concept of a set (grade 5).
4. Calculating the sum of the lengths of intervals removed from the initial segment (grade 6).
5. Generalizing knowledge about the construction of the Cantor set (grade 7).
6. Getting acquainted with infinite and finite sets. Cardinality of the continuum (grade 8).
7. Finding points of the Cantor set using geometric progression (grade 9).
8. Getting acquainted with the Cantor function (grade 10).
9. Calculating the total length of intervals removed when constructing the Cantor set (grade 10).
10. Generalizing knowledge about the Cantor set (grade 11).

It is clear that basic knowledge about Cantor sets is best studied primarily through various forms of informal education, such as club activities, electives, student research projects, and others.

Based on the above, we draw the following conclusions.

Conclusions and prospects for further research. The results of the study showed that it is possible to construct Cantor sets based on number systems with the basis $q=2n+1$ ($n\in N$), as well as Cantor functions on these sets. The article describes and justifies the main points of such constructions. The corresponding constructions can be the basis for practical tasks in the course of function theory and functional analysis in mathematical specialties of higher education institutions.

Analysis of the content of mathematics curricula for specialized classes with in-depth study of mathematics in general secondary education institutions indicates the possibility of studying the topic "Cantor set and number systems" in various types of informal education, during grades 5–11.

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МЕТОДИЧНІ ОСОБЛИВОСТІ ДОСЛІДЖЕННЯ МНОЖИН, ЩО МАЮТЬ ПОТУЖНІСТЬ КОНТИНУУМУ

Анотація. *Проблема. Студенти математичних спеціальностей у вищих навчальних закладах стикаються з канторовими множинами під час вивчення структури замкнутих множин у курсах теорії функцій та функціонального аналізу. У класичних підручниках для визначення потужності цих множин використовується трійкова система числення.*

У цій статті представлено метод визначення потужності канторових множин за допомогою альтернативних систем числення. Такий підхід інтегрує вивчення канторових множин у теорії функцій та функціональному аналізі з системами числення, що розглядаються в курсах алгебри та програмування.

Мета. Метою цієї роботи є вивчення множин, подібних до традиційної досконалої множини Кантора та функції Кантора, з подальшим обґрунтуванням та розробкою плану інтеграції теми "Множина Кантора та системи числення" в курс теорії функцій та функціонального аналізу.

Методи дослідження. При встановленні результатів дослідження автори використовували метод аналізу попередніх публікацій з методики окремих розділів функціонального аналізу та теорії множин у закладах вищої освіти. Використовувався, також, метод аналітичних перетворень, оснований на представленні дійсних чисел у різних системах числення.

Основні результати дослідження. У цій статті представлено метод визначення потужності канторових множин за допомогою альтернативних систем числення. Цей підхід інтегрує вивчення канторових множин у теорії функцій та функціональному аналізі із системами числення, що викладаються в курсах алгебри та програмування.

Наукова новизна результатів дослідження. Вперше потужність канторової множини встановлено за допомогою п'ятіркової системи числення, і показано, що для цього можна використовувати систему числення з непарною основою.

Висновки та пропозиції. Результати дослідження показали, що можливо будувати канторові множини на основі систем чисел з непарним базисом. Відповідні конструкції можуть бути основою для практичних завдань у курсі теорії функцій та функціонального аналізу в математичних спеціальностях вищих навчальних закладів.

Аналіз змісту навчальних програм з математики для спеціалізованих класів з поглибленим вивченням математики в закладах загальної середньої освіти вказує на можливість вивчення теми «Канторові множини та системи чисел» у різних видах неформальної освіти, протягом 5-11 класів.

Ключові слова: *методичні особливості; система числення; множина; потужність множини; канторівська множина; метричний простір; фрактали; функціональний аналіз.*

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IMPLEMENTING CLIL PRINCIPLES IN TERTIARY EDUCATION: A METHODOLOGICAL DIMENSION

Introduction. This article operationalizes a framework for implementing Content and Language Integrated Learning (CLIL) within University methodological practice, specifically for Master's-level learners in Methods of Teaching the English Language to High School and University students. It elucidates the foundational CLIL principles for Foreign Language Pedagogy, examines key CLIL tenets, including instructor mediation, scaffolding, and distinct language layers (subject-specific, general academic, and peripheral), alongside major strategies such as concept mapping. A special emphasis is placed on the incremental progression from Basic Interpersonal Communication Skills (BICS) to Cognitive Academic Language Proficiency (CALP).

The purpose of the article is to illustrate how the elucidated CLIL principles can be effectively implemented into methodological practice.

Results. Employing the methods of theoretical positioning, pedagogical observation, quantitative and qualitative research, and comparative analysis, the article illustrates how CLIL conceptual positions may be applied to methodological classrooms presuming accumulation and progression in the course of foreign language and content acquisition, knowledge discovery, and assimilation of both BICS and CALP.

Conclusion. CLIL curricula must delineate distinct language layers – subject-specific, general academic, and peripheral – each with unique characteristics and functions, necessitating differentiated pedagogical strategies. At that, recognizing the intrinsic link between cognition and language within subject domains is paramount. The progression from BICS to CALP is a protracted process, requiring adherence to a structured procedure facilitated by CLIL instructors. CLIL is underpinned by six key principles that guide teacher scaffolding and framework implementation within CLIL learning environments.

Keywords: *Content and Language Integrated Learning; Foreign Language Pedagogy; Basic Interpersonal Communication Skills; Cognitive Academic Language Proficiency; Scaffolding.*

Introduction. Content and Language Integrated Learning (CLIL) represents a pivotal pedagogical paradigm shift, particularly within the context of tertiary methodological instruction. Defined as a dual-focused educational approach, CLIL systematically integrates the acquisition of disciplinary knowledge with the simultaneous development of an additional language, thereby establishing a symbiotic relationship where the language serves as the medium for content mastery.

While the foundational principles of CLIL are widely recognized, their actualization and implementation within the specialized environment of methodological classrooms presents a distinct set of pedagogical and didactic challenges.

This study introduces the critical need to examine the concrete implementation of CLIL core tenets in settings focused on subject-specific teaching methods, professional practice, and critical curriculum design. Such methodological classrooms necessitate an intricate balance: instructors must not only facilitate the learning of content (e.g., specific research methods, teaching techniques, curricular analysis) but also intentionally cultivate the specialized academic register and communicative and cognitive competence required for students to articulate this knowledge effectively in the target language.

The goal of the article. This research is aimed at actualizing a potential framework for implementing CLIL within University methodological practice. To this end, the present article will elucidate the foundational CLIL principles pertinent to Foreign Lan-